Spectroscopic Signatures of Massless Gap Opening in Graphene

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Gap opening in graphene is usually discussed in terms of a semiconducting-like spectrum, where the appearance of a finite gap at the Dirac point is accompanied by a finite mass for the fermions. In this letter we propose a gap scenario from graphene which preserves the massless characters of the carriers. This approach explains recent spectroscopic measurements carried out in epitaxially-grown graphene, ranging from photoemission to optical transmission.

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In the last few years the realization of free-standing single layers of graphene opened a new route to the possibility of investigating the behavior of massless Dirac fermion in two dimensions. Indeed, in graphene the valence and conduction bands are formed by the p_z orbitals of the carbon atoms arranged on the two sublattices A and B of the honeycomb lattice. When the two sublattices are electrostatically equivalent the two bands meet at the K (K$'$) points of the Brillouin zone, leading to a zero-gap semiconductor with two conical bands $\varepsilon_k^\pm = \pm v_\mathrm{F} k$ ($k = |\mathbf{k}|$), which resemble relativistic Dirac carriers with zero mass and Fermi velocity v_F . For this reason, K and K$'$ are usually referred to as Dirac points. Even though the massless Dirac spectrum makes graphene the perfect playground for investigating relativistic effects in quantum systems, for device application a tunable-gap semiconducting behavior would be more suitable. Along this perspective, of remarkable interest are some recent experiments of Angle Resolved PhotoEmission Spectroscopy (ARPES) in epitaxially-grown graphene, which reveal a finite band splitting $2\Delta \sim 0.26$ eV at the K point [1, 2]. In these works the authors propose that the splitting arises from the inequivalence of the A and B sublattices, which in turn leads to a massive (ms) gapped spectrum [3, 4]:

$$E_{k,\pm}^{\mathrm{ms}} = \pm\sqrt{(v_\mathrm{F} k)^2 + \Delta^2}. \qquad (1)$$

The magnitude of the band-splitting gap has been reported to decrease by increasing the number of layers, or, from another perspective, by reducing the induced charge density. Similar ARPES spectra were reported previously in Ref. [5], although a different interpretation was proposed[5, 6, 7]. The most convincing argument in such a controversy comes from the electronic dispersion far from the Dirac point, which is at odd with the massive gapped spectrum of Eq. (1). Indeed, Eq. (1) predicts $E_{k,\pm}^{\mathrm{ms}}$ to be unaffected by the gap opening in the $|E_{k,\pm}^{\mathrm{ms}}| \gg \Delta$ regime, and in particular the linear asymptotic behavior of the upper band $E_{k,+}^{\mathrm{ms}} \approx v_\mathrm{F} k$ ($E_{k,+}^{\mathrm{ms}} \gg \Delta$) should match the linear behavior of the lower band $E_{k,-}^{\mathrm{ms}} \approx -v_\mathrm{F} k$. A careful analysis of the ARPES data reveals on the contrary a finite off-shift of the two asymptotic linear behaviors [1, 2, 5, 6], which cannot be explained even by the periodic modulation induced by the substrate [8]. Further discrepancies appear also in the profile of the dispersion at finite k_x away from the K point (Ref. [1], Fig. S3): instead of the parabolic shape predicted by Eq. (1), a "V" shaped conic profile appears, characteristic of a massless dispersion.

Quite remarkably, the issue of the gap opening has been raised also by very recent optical-absorption measurements in epitaxial graphene[9]. Indeed, the analysis of the optical spectra based on the massive gap model (1) has two major drawbacks: from one side, it would suggest that no gap (or a negligible one) is present in the system, in contrast with ARPES results, from the other side it fails in reproducing the data in the visible frequency range, where the "universal" conductivity value of $e^2/4\hbar$ is expected theoretically[10, 11, 12], and tested experimentally in multi-layer graphite samples[13] or few-layers suspended graphene[14]. In this Letter, we propose a different gap scenario that reconciles the gapped nature of the spectrum with the massless character of the fermions in graphene. As we shall see, such a gap model not only accounts very well for the ARPES spectra, but reconciles also optical measurements performed from the visible to the far-infrared (IR) regime.

Let us start by introducing the Hamiltonian for free electrons in the graphene honeycomb lattice, in terms of the usual spinor $\psi_\mathbf{k}^\dagger = (c_{\mathbf{k},\mathrm{A}}^\dagger, c_{\mathbf{k},\mathrm{B}}^\dagger)$. Linearizing around the K Dirac point (we put $\hbar = 1$), and using $\mathbf{k} = k(\cos\phi, \sin\phi)$, we can write:

$$\hat{H}_\mathbf{k}^0 = v_\mathrm{F} k \begin{pmatrix} 0 & \mathrm{e}^{-i\phi} \\ \mathrm{e}^{i\phi} & 0 \end{pmatrix}, \qquad (2)$$

whose eigenvalues are the usual gapless Dirac cones $\varepsilon_k^\pm = \pm v_\mathrm{F} k$. If the A and B sublattices are electrostatically inequivalent, an additional term $\propto \Delta\hat{\sigma}_z$ ($\hat{\sigma}_{i=I,x,y,z}$ being the Pauli matrices) should be added to $\hat{H}_\mathbf{k}^0$, and the gapped spectrum of Eq. (1) is recovered. In similar way, in Ref. [4] it was shown that a gap opening, related to inter-valley scattering, is always associated with a mass onset. Finally, off-diagonal intra-valley processes of the kind $Q_1^x\hat{\sigma}_x + Q_1^y\hat{\sigma}_y$ were claimed to lead to

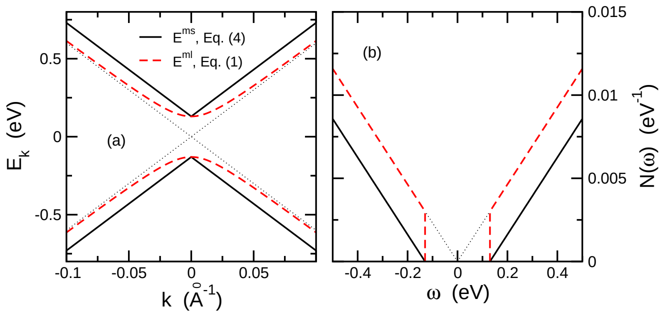

FIG. 1: (Color online). Electronic dispersion (a) and DOS (b) for the massless $E^{\rm ml}$ (solid black line) and massive $E^{\rm ms}$ (dashed red line) gapped models, with $\Delta = 0.13$ eV. The dotted line refers to the Dirac-like dispersion ε in the absence of any gap.

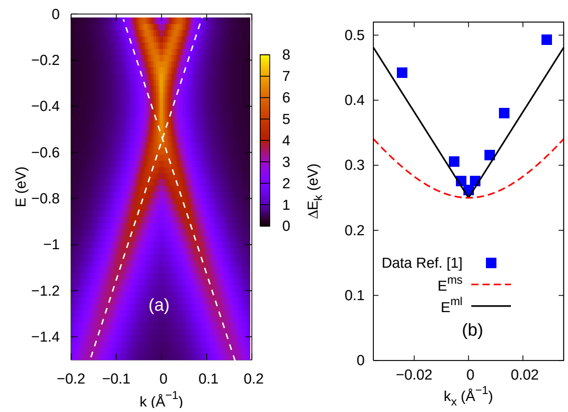

FIG. 2: (color online). (a) Intensity map of the spectral function $A(k,\omega)$ of the massless model [Eq. (4)] for $\Delta = 0.13$ eV, $v_F = 6$ eV Åand Γ taken from the experiments (see text). Dashed white lines show the mismatch between the asymptotic linear behavior of the upper and lower bands. (b) Plot of the gap edge at finite k_x for the massless and massive gap model (Δ, v_F as in panel (a)). Dark squares are experimental data taken from Ref. [1].

a mere displacement of the Dirac cone off the K (K′) points [4]. However, this statement is correct only as far as a constant value of Q_1^x, Q_1^y for $\mathbf{k} \to 0$ is assumed, while in general a momentum dependence of the off-diagonal self-energy cannot be ruled out. For instance, the (unscreened) Coulomb interaction leads to the off-diagonal self-energy $\hat{\Sigma}_{\mathbf{k}} \propto k \log(k_c/k)[\cos\phi\hat{\sigma}_x + \sin\phi\hat{\sigma}_y]$, where k_c is a momentum cut-off for the Dirac-like conical behavior[15]. In this case, a correction to the linear Dirac dispersion is achieved, but no gap opens because $\hat{\Sigma}$ vanishes as $k \to 0$.

In this Letter we want to explore a somehow intermediate possibility, where the off-diagonal self-energy has the structure:

$$\hat{\Sigma}_{\mathbf{k}} \simeq \Delta[\cos\phi\hat{\sigma}_x + \sin\phi\hat{\sigma}_y]. \tag{3}$$

It is straightforward to check that, in this case, a gap is opened at the K (K′) points without the onset of a massive term. Indeed, Eq. (3) leads to the massless (ml) gapped spectrum of the form:

$$E^{\rm ml}_{k,\pm} = \pm(v_{\rm F}k + \Delta), \tag{4}$$

with a corresponding density of states (DOS):

$$N(\omega) = \frac{V_{\rm BZ}}{2\pi v_{\rm F}^2}(|\omega| - \Delta)\theta(|\omega| - \Delta), \tag{5}$$

where $V_{\rm BZ} = 5.24$ Å^2 is the volume of the Brillouin zone.

While the main aim of this Letter is to focus on the phenomenological outcomes of this spectrum, a brief discussion on the microscopic origin of the self-energy (3) will be given later. In Fig. 1 we compare the electronic dispersion (panel a) and the DOS (panel b) of the models (4) and (1). Two main striking features need here to be stressed: *i*) the gap in Eq. (4), as mentioned, is created without affecting the conical electronic dispersion. In this sense no massive term is induced; *ii*) in contrast to the case of the massive gap model (1), the lower and upper cones of the spectrum (1) are misaligned by the quantity 2Δ. Such anomalous features are reflected in the DOS [Eq. (5)] where the linear vanishing of $N(\omega)$ as $|\omega| \to \Delta^+$ in the model (4) points out the absence of a quadratic massive term, while the misalignment of the cones is reflected in a corresponding mismatch of the linear dependence of the DOS, which does not extrapolate to zero at the Dirac point.

To make a comparison with the ARPES results in epitaxially-grown graphene, we show in Fig. 2a the calculated spectral intensity $I(\mathbf{k},\omega) = A(\mathbf{k},\omega)f(\omega)$ of the massless model $E^{\rm ml}_{\pm}$, where μ is the chemical potential, $f(\omega)$ is the Fermi function and $A(\mathbf{k},\omega)$ is the spectral function $A(\mathbf{k},\omega) = -{\rm Tr}\{{\rm Im}[(\omega+\mu+i\Gamma)\hat{I} - \hat{H}^0_{\mathbf{k}} - \hat{\Sigma}_{\mathbf{k}}]^{-1}\} = \Gamma/(2\pi)\{[(\omega-\mu-v_F k-\Delta)^2+\Gamma^2]^{-1} + [(\omega-\mu+v_F k+\Delta)^2+\Gamma^2]^{-1}\}$. In Fig. 2 we used $\mu = 0.4$ eV, $2\Delta = 0.26$ eV ($\Delta = 0.13$ eV)[1] and we assumed a quasi-particle scattering rate $\Gamma(\omega) = \Gamma_0 + \alpha|\omega|$, with $\Gamma_0 = 0.165$ eV and $\alpha = 0.11$ fitted from the ARPES data[2] away from the Dirac point using $v_{\rm F} = 6$ eV Å[16]. Similar values could be estimated from Ref. [5]. Notice that the presence of such a large scattering time at the Dirac point partly spoils the gap feature as $k \to 0$, because the two peaks of $A(\mathbf{k} = 0,\omega)$ at $\omega = \pm\Delta$ overlap and cannot be resolved, leading to the controversial interpretation of similar ARPES spectra in Refs. [5, 6] and [1, 2], respectively. The same feature is observed in Fig. 2a, where the exact dispersion at the Dirac point is not detectable due to the large electronic damping. Nevertheless, one can still clearly resolve the net misalignment between the upper and lower Dirac cones, which is peculiar to the model (4), and it is in perfect agreement with all the existing experimental data [1, 2, 5]. Moreover, ARPES data taken away

from the K point provide us also with a direct evidence of the linear (massless) behavior of the electronic dispersion at finite k. By measuring the dispersion at $k_y = 0$ and finite k_x, as extracted from several k_y cuts through the K point, one can easily discriminate between the two models (1) and (4). Indeed, within the model (1) the energy spectrum $\sqrt{\Delta^2 + (v_F k_x)^2}$ would appear parabolic within a momentum window $k_x \lesssim \Delta/v_F \simeq 0.02$ Å^{-1}, while within the massless model (4) one expects a linear increase of the gap, $\Delta + v_F|k_x|$. The experimental data by Ref. [1] (Fig. S3 in Supplementary material) are reported in Fig. 2b: as one can see, the massive model $E^{\rm ms}_{\pm}$ shows no resemblance with the data, while the massless model $E^{\rm ml}_{\pm}$ allows one an excellent fit of the dispersion without any adjustable parameter.

The massless character of the spectrum has also significant consequences on the structure of the optical conductivity $\sigma(\Omega)$. To elucidate this issue we evaluate here the optical conductivity in the bare-bubble approximation. As usual, $\sigma(\Omega)$ is given by two parts, associated to intraband and interband transitions[10, 11]. At the leading order in $\Gamma/|\mu|, \Gamma/\Delta \ll 1$, we obtain:

$$\begin{aligned}
\sigma_{\rm intra}(\Omega) &= -\frac{e^2}{\pi\hbar}\frac{2\Gamma}{\Omega^2+4\Gamma^2} \\
&\quad \times \int d\omega \frac{df(\omega-\mu)}{d\omega}\left(|\omega|-\Delta\right)\theta[|\omega|-\Delta] \\
&\underset{T\to 0}{\approx} \frac{e^2}{\hbar}\delta(\Omega)\left(|\mu|-\Delta\right)\theta[|\mu|-\Delta], \qquad (6)\\
\sigma_{\rm inter}(\Omega) &= \frac{e^2}{\pi\hbar}\int d\omega \frac{f(\omega-\mu)-f(\omega+\Omega-\mu)}{4\Omega} \\
&\quad \times \frac{2\Gamma}{(\omega+\Omega/2)^2+4\Gamma^2}\left(|\Omega|-2\Delta\right)\theta\left[|\Omega|-2\Delta\right] \\
&\underset{T\to 0}{\approx} \frac{e^2}{4\hbar}\left(1-\frac{2\Delta}{|\Omega|}\right)\theta\left[|\Omega|-2\max\left(\Delta,|\mu|\right)\right]. \qquad (7)
\end{aligned}$$

As one can see, the general structure of the optical conductivity is the same for all the gapped and ungapped models: a Drude peak of width $\Gamma_{\rm opt} = 2\Gamma$ and an interband contribution which starts at a threshold given by the larger between 2μ and 2Δ, and saturates at $\Omega \gg \Delta, |\mu|$ to the universal value $e^2/4\hbar$ [10, 11, 12]. When a gap opens, part of the spectral weight is transferred to the interband transitions and the Drude peak decreases with respect to the ungapped case. In the model (1), $\sigma(\Omega)$ can be obtained by replacing the factors $(|\mu| - \Delta) \to (|\mu| - \Delta^2/|\mu|)$ and $(1 - 2\Delta/|\Omega|) \to (1 + 4\Delta^2/|\Omega|^2)$ respectively in Eqs. (6) and (7)[10, 11]. Two features allow thus one to differentiate the models (1) and (4): the relative weight of interband and intraband contributions and the shape of the conductivity at the interband threshold. In particular, one can see that (*i*) the gap-induced reduction of the Drude peak is much stronger in the massless gap model (of order $\sim \Delta$) than in the massive one (of order $\sim \Delta^2/|\mu|$), see inset of Fig. 3a); (*ii*) while the massive gap model would give rise

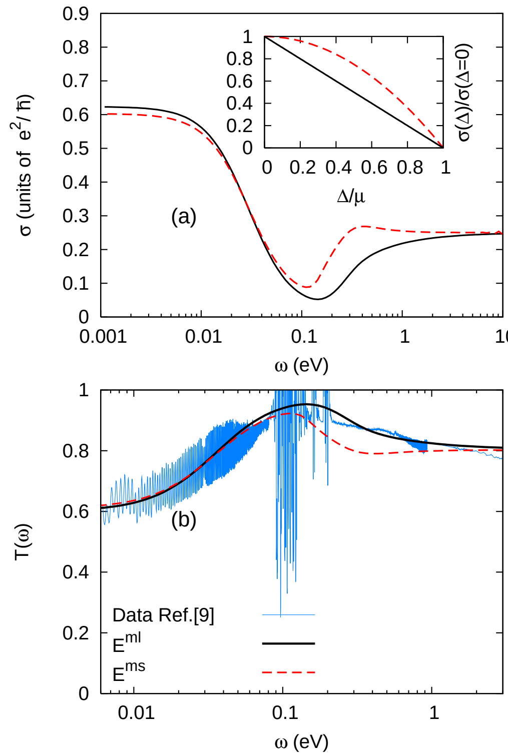

FIG. 3: (Color online). (a) Optical conductivity $\sigma(\Omega)$ for $\mu = 0.1$ eV, $T = 300$ K, $\Gamma = 15$ meV for the massless gap model (solid black line, $\Delta = 45$ meV) and for the massive gap model (dashed red line, $\Delta = 73$ meV). Inset: reduction of the zero-frequency optical conductivity in the two cases, as a function of Δ/μ. (b): Comparison of the two models with the experimental data for $T(\omega)$ from Ref. [9].

to a *peak* above the asymptotic value $e^2/4\hbar$ at the edge of the interband spectrum and to a rapid saturation to the asymptotic value, the massless model predicts a *depletion* in the correspondence of such edge (due to the factor $1 - 2\Delta/\Omega < 1$), followed by a slow saturation to the universal value, see Fig. 3a. Such a depletion is again a consequence of the vanishing of the DOS at the gap in the massless model (see Fig. 1b), with consequent reduction of the spectral weight for transitions occurring between the two bands.

The present results open thus a new perspective in the analysis of the optical properties of graphene grown epitaxially on SiC substrates, that can be deduced from measurements of the optical transmission $T(\omega)$, which in first approximation is related to the real part of the single-layer optical conductivity as $T(\omega) = [1 + N\sigma(\omega)\sqrt{\mu_0/\epsilon_0}/(1 + n_{\rm SiC})]^{-2}$[9, 12], where $n_{\rm SiC}$ is the refractive index of SiC and N is the number of layers. In Ref. [9], indeed, transmission data for few-layers graphene samples in the frequency range from the far-IR to the visible were analyzed within the context of the massive gap model described in Eq. (1), and the gap was concluded to be negligible within the experimental accu-

racy $\Delta \lesssim \Gamma_{\rm opt} \simeq 10$ meV [9]. Such a fit reproduces the data in the far-IR to mid-IR range, but fails in the visible range, where, according to this fit, the data would then indicated a conductivity *larger* than the universal value $e^2/4\hbar$. The failure of the fit follow from the fact that the size of optical transmission $T(\Omega)$ at $\Omega \approx 3$ eV and for $\Omega \to 0$, and hence the size of the optical conductivity in the corresponding range, are found to be of the same magnitude. According to the previous discussion, this could be achieved by a transfer of spectral weight from the Drude peak to the interband conductivity, as due to a gap opening. However, to reproduce this feature within the massive gap model one would need $\Delta \approx |\mu|\sqrt{1-\pi\Gamma/2|\mu|} = 87$ meV, where $|\mu| \simeq 0.1$ eV, $\Gamma = \Gamma_{\rm opt}/2 \approx 15$ meV are extracted from the interband edge and from the width of the Drude peak [9]. With such large value, $\Delta \gg \Gamma$, the massive gap would be clearly detectable as a sharp peak at the interband edge, which is instead absent in the data. For this reason, the authors of Ref. [9] extracted a vanishing gap from the fit based on the model (1), which then fails in reproducing the data in the visible range.

Such ambiguity can be naturally solved within the context of the massless gap model where: *i*) a relatively smaller gap value is needed to make the low and high frequency optical conductivity of the same magnitude; *ii*) the opening of a massless gap does not give rise to any peak at the threshold of interband transitions but to a *depletion* of the conductivity with respect to the universal value, and then to a slow and smooth crossover toward the high-frequency regime, as observed in the data. This picture is in very good agreement with the actual experimental measurements. In Fig. 3b we show the best fit to one set of the experimental data of Ref. [9] by using the massless gap model. We take $|\mu| = 0.1$ eV, $T = 300$ K and $\Gamma = 15$ meV from the experiments themselves and we estimate $\Delta = 45$ meV, $N = 18$. For comparison, the fit with the massive gap model, constrained to reproduce the experimental values of $T(\omega)$ in the low and high frequency limit, would give $\Delta = 73$ meV and $N = 18$, and would result in a clear peak (shoulder) at the interband edge. We would like to stress that the even though the absence on an interband peak can also be accounted for by a vanishing gap, the similar magnitude of the low and high-frequency values of the optical transmission is a clear indication of a reduced Drude height, and hence of the presence of a gap.

So far, we have analyzed in details the consequences on ARPES and optical spectra of the massless gap induced by a self-energy as described in Eq. (3). In the last part of this Letter we discuss the possible physical origin of such off-diagonal self-energy. To this aim we expand the electron Green's function and the self-energy in terms of their Pauli components where, for the case of equivalent carbon sublattice, we can neglect the $\hat{\sigma}_z$ term. In the noninteracting case the off-diagonal Green's function, for instance $\propto \hat{\sigma}_x$, has the simple form: $G_x(\mathbf{k}, i\omega_n) = -k\cos\phi/[(i\omega_n+\mu)^2 - v_{\rm F}^2 k^2]$. At the Hartree-Fock level, one can quite generally write $\hat{\Sigma}_{\mathbf{k}} = T\sum_{\mathbf{k}',n} V(\mathbf{k}-\mathbf{k}')\hat{G}(\mathbf{k}, i\omega_n){\rm e}^{i\omega_n 0^+}$, which can have off-diagonal components depending on the form of the potential $V(\mathbf{k}-\mathbf{k}')$ (see for example the case of unscreened Coulomb interaction discussed in Ref. [15]). While a completely momentum-independent interaction would thus lead to a vanishing off-diagonal contribution, due to the angular average, a small anisotropy in the scattering angle can lead to a self energy of the form (3). For example, one can use $V(\mathbf{k}-\mathbf{k}') = V\theta(|\phi-\phi'|-\phi_c)$, where $\phi_c = \pi$ corresponding to isotropic scattering. In this case, one can easily check that $\hat{\Sigma}_{\mathbf{k}}$ has exactly the form (3), with $\Delta = V^2 k_c^2 \sin^2\phi_c/8\pi^2$. Further investigation on the possible source of anisotropic scattering could thus shed more light on the microscopic origin of the proposed self-energy (3). In this context, it is worth noting that a possible role of the doping is suggested by the observed vanishing of both the gap and the band misalignment as a function of the charge concentration[1, 5].

In summary, we propose a gapped model for graphene which allows one to reconcile the massless Dirac character of the carriers with the effects related to a gap opening at the Dirac point. We show that both ARPES and optical-conductivity measurements give clear indications of such massless gap opening in epitaxially-grown graphene. Since ARPES measurements are available only for epitaxially-grown graphene, we cannot rule out the possibility that such a gap opening is restricted to these systems. Recent tunneling data on epitaxially-grown[17] and suspended graphene [18] are not conclusive on this respect: however, our predictions can be further tested experimentally and, if confirmed, they would pose stringent constraints on the interaction mechanisms at play in graphene.

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[1] S. Y. Zhou *et al.*, Nat. Mat. **6**, 770 (2007).
[2] S.Y. Zhou *et al.*, arXiv:0801.3862 (2008).
[3] V.P. Gusynin, S.G. Sharapov, and J. P. Carbotte, Int. J. Mod. Phys. B **21**, 4611 (2007).
[4] J. L. Mañes, F. Guinea, and M. A. H. Vozmediano, Phys. Rev. B **75**, 155424 (2007).
[5] A. Bostwick *et al.*, Nat. Phys. **3**, 36 (2007).
[6] A. Bostwick *et al.*, New. Jour. of Phys. **9**, 385 (2007).
[7] E. Rotenberg *et al.*, Nat. Mat. **7**, 258 (2008); S. Y. Zhou *et al.*, Nat. Mat. **7**, 259 (2008).
[8] S. Kim *et al.*, arXiv:0712.2897 (2007).
[9] J.M. Dawlaty *et al.*, arXiv:0801.3302 (2008).
[10] T. Ando, Y. Zheng, and H. Suzuura, J. Phys. Soc. Jpn.

71, 1318 (2002).
[11] V. P. Gusynin, S. G. Sharapov, and J. P. Carbotte, Phys. Rev. Lett. **96**, 256802 (2006).
[12] T. Stauber, N. M. R. Peres, and A. K. Geim, arXiv:0803.1802 (2008).
[13] A. B. Kuzmenko *et al.*, Phys. Rev. Lett. **100**, 117401 (2008).
[14] R.R. Nair *et al.*, arXiv:0803.3718 (2008).
[15] E. G. Mishchenko, Phys. Rev. Lett. **98**, 216801 (2007).
[16] A.H. Castro Neto *et al.*, arXiv:0709.1163 (2007).
[17] L. Vitali *et al.*, preprint 2008.
[18] G. Li, A. Luican and E. Y. Andrei, arXiv:0803.4016 (2008).